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TIDE-LEVEL AND BOTTOM FRICTION EFFECTS ON WAVE REFRACTION AS DETERMINED BY NUMERICAL WAVE REFRACTION PROCEDURES

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TIDE-LEVEL AND BOTTOM-FRICTION EFFECTS
ON WAVE REFRACTION AS DETERMINED BY
NUMERICAL WAVE REFRACTION PROCEDURES

by

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Submitted in partial fulfillment of the requirements for the degree of

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NAVAL POSTGRADUATE SCHOOL June 1967 NPS ARCHIVE 196 FARRELL, C

ABSTRACT

Numerical wave refraction programs permit a detailed study of the transformation of wave energy as waves move from deep water to shallow water. By eliminating the subjectivity that is present with hand drawn diagrams the effect of small variations in the initial assumptions and wave conditions can be investigated. The effects on wave refraction of tide level changes and bottom friction are investigated here. It is demonstrated that a uniform increase in water level as would be caused by tidal fluctuations can cause a significant change in the wave refraction pattern for a given nearshore region. A computing procedure is developed to permit numerical refraction programs to account for bottom friction. The reduction in wave height caused by bottom friction depends primarily on bottom slope; this relation is shown in tabular and graphical form.

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TABLE OF SYMBOLS

- A approach angle of wave ray
- $oldsymbol{eta}$ ray separation in shallow water divided by ray separation in deep water
- C wave velocity
- Co deep water wave velocity
- f dimensionless friction coefficient for the bottom.
- FK curvature of ray
- g acceleration due to gravity
- H wave height at position x
- Ho deep water wave height
- H_1 wave height at position x_1
- H/Ho wave height at position x divided by deep water wave height
- HHFR wave height at position x, when bottom friction is included, divided by deep water wave height
 - ${\rm K}_{\rm f}$ wave height reduction factor due to bottom friction alone
 - K_r refraction factor
- $K_{\mbox{fr}}$ combined friction and refraction factor
 - K_s direct shoaling factor
 - L wave length
 - Lo deep water wave length
 - s wave ray
 - T wave period
 - x distance measured along the wave ray in the direction of propogation of the waves
- △ x x x₁
 - γ 3.1416...

Note: All values are assumed to be in the English System of Units.



1. Introduction

Preparing accurate wave refraction diagrams is a necessary first step in many coastal engineering and amphibious operations. The advent of the numerical wave refraction programs [Griswold (1963), Stouppe (1966), Wilson (1966)] has considerably reduced the time required to construct a wave refraction diagram. The speed of the computer operations permits more parameters to be included in the calculations and improves the accuracy of the output. Of equal importance is the fact that numerical products provide a completely objective basis for evaluating the cumulative effect of various assumptions on wave refraction.

The present study has investigated the effects of tide-level changes and bottom friction on wave-refraction patterns for several environments. The investigation first compared the wave refraction programs of Griswold (1963), Stouppe (1966), and Wilson (1966). Since there was no way available to the author to compare program accuracy on an absolute scale, the comparison was based on method of computation, simplicity of use, and presentation of results.

The program written by Stouppe was considered to have the most potential and was chosen for use in this investigation. The major advantages of Stouppe's program are these:

- (1) The program uses water depth directly as the interpolation surface for evaluation of wave speed, thereby limiting the number of computations and increasing the program versatility.
- (2) The program uses a second-order non-linear differential equation to determine the refraction coefficient at each point along the ray. Thus, the wave height at each point can be computed.

- (3) The program uses constant time steps in the computation rather than constant distance steps. This reduces the distance between computations as the shoreline is approached. Thus for the same number of computer operations a greater percentage of computations are in the shallowest area where refraction, friction, etc., are most important, and are changing rapidly.
- (4) The program includes a graphical output that plots the wave orthogonals, wavecrests and the shoreline.

The following modifications were made to Stouppe's program:

- (1) The program was made to recycle, permitting more effective use of computer time when more than one diagram is to be constructed.
 - (2) Provision was made to add a constant value to the depth field to investigate tide level effects.
- (3) A subroutine was added to account for bottom friction in the computation of wave height.

2. Computer Program.

Depth grid.

The first step in the utilization of this program is the construction of a grid of water-depth values for the desired area. The grid must be sufficiently large so that the starting point of all rays to be followed is in deep water, i.e., the ratio of water depth to deep water wave length (d/ $L_{\rm O}$) is greater than 0.5. By convention the x-axis is positive increasing toward the shore while the y-axis is positive to the left of the x-axis. The grid interval is selected so that the bottom contours are reasonably parallel to one another within a given square.

In constructing the depth grid it is advantageous to record the average value of all depths within that grid square, rather than the value of water depth at the intersection point. This procedure tends to correct for small variations in the depth of water within the depth grid without necessitating the use of a smaller grid size. All actual depths are made positive; extrapolated depth values are continued on land for two grid units from the shoreline and are made negative. Beyond two grid units from the shoreline any arbitrary negative depth may be used. For depths on the shoreline itself zero is used.

To reduce the time required to obtain the depth grid a clear plastic overlay with a black dot at each grid intersection point was prepared by initially locating the grid dots on white paper and producing a viewfoil transparency with an Ozalid copier, Model 400.

Input data.

Stouppe's order of data input was modified to permit the program to recycle. The present data deck consists of three sections. The first section contains only the value of MM and NN which designate the number of points in the depth grid in the x and y directions respectively. The

second section contains the water depth grid. These values are arranged so that the computer reads all the y values for each successive x-grid position. Water depths are read in fathoms and are converted to feet within the computer program. The last section contains one card for each set of input parameters for which it is required to construct a wave refraction diagram. The input parameters include:

X,Y: The grid position for the starting point of the first ray (feet). All starting rays must be more than two grid intervals in distance from the edge of the depth grid.

FCF: The coefficient of bottom friction.

TIDE: The tide level, in feet, above chart depth for which computation is required. This value is subtracted from the depth grid after each refraction pattern is completed.

HINT: The deep water wave height (feet). This is required for the friction subroutine.

T: The initial wave period (seconds).

Al: The initial wave angle (degrees). The smaller angle measured between the positive Y direction and the initial wave crest. The angle is made negative if the slope of the wave crest is negative.

NOR: The number of rays to be followed.

TIME: The time interval between computations for advancing of the wave front (seconds).

DIST: The distance between rays (feet).

GRID: The grid interval (feet).

This order of data input permits wave refraction diagrams to be computed for many combinations of input variables while requiring the computer to assemble the program and read the depth field only once.

Computer output.

The output from the computer consists of both a printed output and a graphical output. The printed output variables are these:

X,Y: The x and y coordinates for a given wave crest and ray number (yards).

COREFR: The coefficient of refraction (K_r) for the wave at the point x,y.

HHO: The ratio of wave height to deep water wave height neglecting bottom friction.

HHFR: The ratio of wave height to deep water wave height when bottom friction is included.

NGO: Indicates that the ray has terminated (1), or it is continuing (2).

DEPTH: The water depth for grid position X and Y (feet).

The graphical plot of wave crests is programmed for the Calcomp 160 system utilizing the DRAW subroutine programmed by J. R. Ward for the Fortran 60 system. See Appendix II, Stouppe (1966), for subroutine listing. The first and each succeeding third wave crest are plotted. The zero water depth points are used for contouring the shoreline on the graph.

Appendix 2 contains a sample of the printed and graphical output for Monterey Bay, California.

Computer operations.

Since the details of the computer operations are discussed by Stouppe (1966), only the main features will be discussed here. The formulas used in the computations are listed in Appendix 3.

The water depth at the first point is computed by fitting the closest nine grid-point depths to a quadric surface by the least-squares method. An iterative procedure is then used to solve for wave velocity. It is assumed here that wave velocity is a function of water depth and period only.

The wave ray is then moved to the next point by solving for the ray curvature and projecting the ray forward a distance equal to the product of the wave speed for the point multiplied by TIME. At this new point the value of ray separation (β), coefficient of refraction (K_r), shoaling factor (K_s), H/H_O, and HHFR are calculated.

This procedure is repeated until all orthogonals have been advanced one time interval. The new wave crest is plotted and the entire procedure is repeated until all orthogonals have either gone off the grid or reached the shoreline.

3. The Effects of Tide Level Changes on Wave Refraction Patterns.

Differences in shoaling factor, wave speed, and ray curvature at specific depths.

The effect on wave refraction of a uniform increase in water depth caused by tide level changes is cumulative and is dependent on the bottom depth contours. Therefore, to evaluate the effect properly, a wave ray must be followed along its entire distance from deep water to the shore. However, by calculating the value of shoaling factor (K_S) , wave speed (C), and ray curvature (FK) at both high tide and low tide it is possible to evaluate the order of magnitude of the differences in the basic parameters of wave refraction caused by tide level fluctuations.

The value of K_S , C, and FK were calculated initially for 25, 50, and 100 foot water depths; second calculations were made for a water depth equal to the initial water depth plus a tide level increase of 6, 10, and 16 feet. To evaluate these differences on a relative scale a percent difference was calculated by using the formula:

% difference
$$X = \frac{\chi_{HT} - \chi_{LT}}{\chi_{HT}}$$
 (1)

where

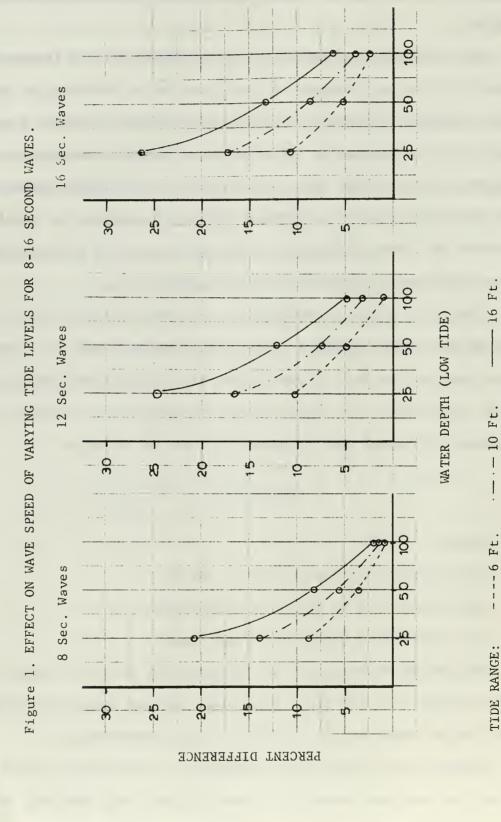
X = variable representing C, K_S , or FK

 X_{ht} = value of X at high tide water depth

 X_{1t} = value of X at low tide water depth.

The percent difference of $K_{\rm S}$ was generally small and under certain conditions went to zero (for those values associated with the inflection point in the curve when $K_{\rm S}$ is plotted against water depth).

Figure 1 shows the percent difference in wave speed corresponding to the low tide water depth for a specific tide range and wave period.



It is apparent that the difference is often greater than 10 percent and increases with increasing wave period, increasing tide range and decreasing low tide water depth.

The values of ray curvature (FK) were determined using the equation:

$$FK = \frac{1}{c} \left[\sin A \frac{\partial C}{\partial x} - \cos A \frac{\partial C}{\partial y} \right]$$
 (2)

where

A = approach angle

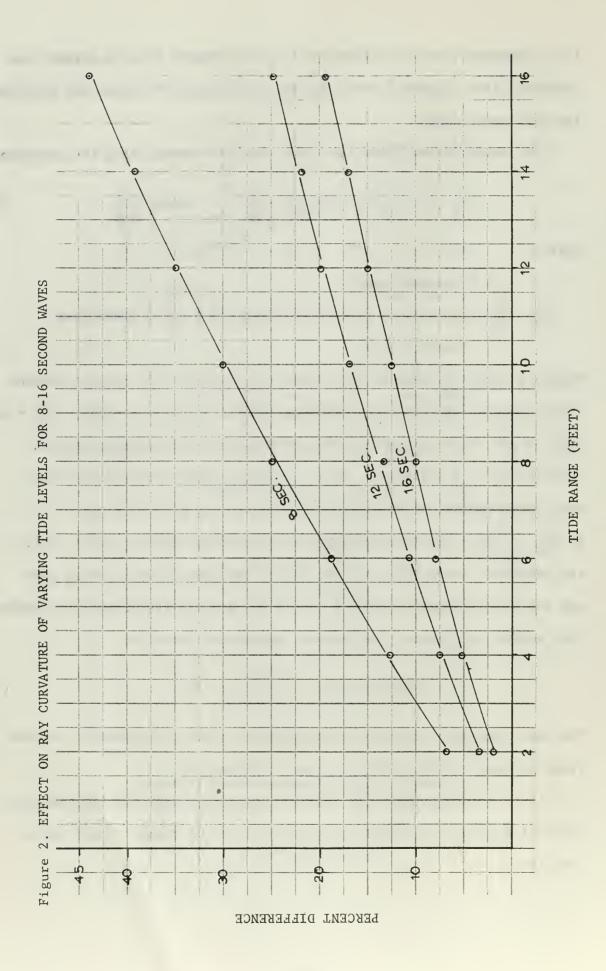
 $\frac{\partial C}{\partial x}$, $\frac{\partial C}{\partial y}$ = derivative of wave speed in the X and Y directions respectively.

Figure 2 shows the percent difference in FK versus tide range (computed using equation (2) with the following initial values: $A = 30^{\circ}$, $\frac{\partial C}{\partial y} = 0$, $\frac{\partial C}{\partial y} = \Delta C$ for a twenty foot change in water depth starting with an initial depth of 100 feet). The values of percent difference plotted are representative of the percent difference for all values of A, $\frac{\partial C}{\partial y}$, $\frac{\partial C}{\partial y}$, and water depth; this occurs because the terms inside the brackets in equation (2) have nearly the same value for high tide and low tide and tend to cancel out in the percent difference calculation. This reduces the percent difference calculation for FK to

% difference FK =
$$\frac{C_{LT} - C_{HT}}{C_{HT}}$$
 (3)

The fact that the difference calculation is primarily dependent on tide range and wave speed is clearly shown in Figure 2.

It is concluded that the change between high tide and low tide in a refraction pattern is primarily determined by the change in ray curvature (FK).



Difference in wave refraction patterns for specific locations.

To investigate the combined effect of the changes in C, $K_{\rm S}$ and FK caused by a uniform increase in water depth, wave refraction diagrams were constructed for several locations using both the low tide water depths and the high tide water depths.

The first area considered was the California coast near Big Sur, between latitude 36° 14′ 30″ and latitude 36° 17′ 42″. This area was chosen because eight-second waves arriving on the coast will be refracted by the offshore bar (shown by the shaded area in Figures 3 and 4) at low tide but not at high tide. Thus tide level changes will cause selective refraction by the offshore bar. Figure 3 shows the refraction pattern for 8 second waves approaching from 270 degrees at low tide; Figure 4 shows a different pattern for the same condition at high tide. 1

Figures 3 and 4 show two major differences in the refraction pattern. First, orthogonals 6 and 7 cross approximately halfway to shore at low tide but they do not cross at high tide. Secondly, at low tide there is a convergence of wave orthogonals at Point A; this convergence is not present at high tide.

¹To make the changes in the orthogonal pattern more prominent Figures 3-8 are plotted with an expanded ordinate scale. Also, the computer was not used to contour the wave crests but rather the wave orthogonals were contoured manually; the shoreline was indicated by a triangle plotted whenever water depth was zero. TIME equalled 30 seconds for all diagrams.

²Interpretation of the effect of crossed orthogonals is beyond the scope of this paper. In this discussion we will only be concerned with the difference in ray paths between low tide and high tide.

Table 1 shows the value of the wave-height ratio (H/H_O) computed at four wave crests before the shoreline is reached for specific points along the coast. This table clearly shows that the value of wave height can be considerably different at low tide and high tide.

Figures 5 and 6 represent the same initial conditions as Figures 3 and 4 except that the initial wave direction is from 310 degrees. The change in the orthogonal pattern is clear. It is noted that in Figures 3 and 4 the primary change in the orthogonal pattern (between low tide and high tide) is in the vicinity of points B and C, whereas in Figures 5 and 6 the greater change occurs near Point A.

Wave refraction diagrams were also computed for the coast of Maine between latitude 44° 39′ 35″ and latitude 44° 44′ 45″. This area was chosen in order to determine if the combination of a large tide range (16 feet) and deep water depths (greater than 200 feet a short distance from the shoreline) would cause a change in the refraction pattern. Figures 7 and 8 show the refraction patterns for 16-second waves arriving from 070 degrees at low tide and high tide. The difference in refraction patterns is evident. Table 3 shows the H/H_O ratios (calculated at four wave crests before reaching the shoreline) for specific locations at low-tide and high-tide conditions.

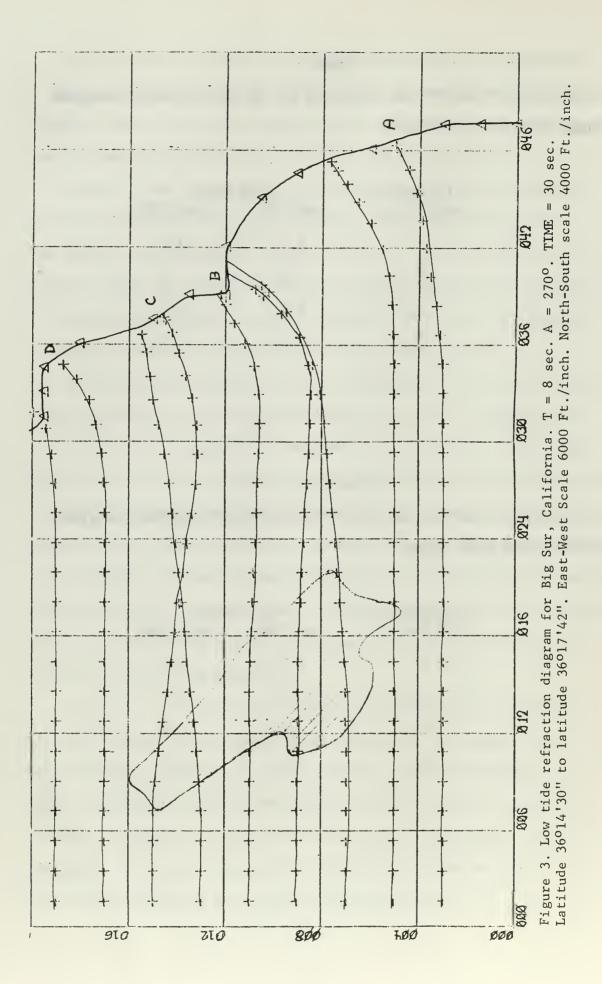
In total, about 20 sets of refraction patterns were analyzed and it appears impossible to predict what changes in wave height and refraction patterns will occur as the result of tide level changes. The only generalization that can be made is that the curvature of an orthogonal is generally greater at low tide than at high tide. It is concluded that precise water-level information, including tide state, should be incorporated into wave refraction procedures.

TABLE I $\label{eq:hho} \text{H/H}_{\text{O}} \text{ RATIO AT LOW TIDE AND HIGH TIDE FOR SPECIFIC COASTAL LOCATIONS}$ NEAR POINT SUR, CALIFORNIA.

| LOCATION (SEE FIG. 3) | $_{ m LOW\ TIDE}^{ m H/H_O}$ | RATIO HIGH TIDE |
|-----------------------|------------------------------|--------------------|
| A | 1.1 | 1.2 |
| В | 1.3 | .9 |
| С | 3.1 | 1.7 |
| D | .8 | .8 |

TABLE II $\mbox{H/H}_{\mbox{\scriptsize o}} \mbox{ RATIO AT LOW TIDE AND HIGH TIDE FOR SPECIFIC COASTAL LOCATIONS}$ NEAR EASTERN HEAD, MAINE.

| LOCATION (SEE FIG. | | RATIO HIGH TIDE |
|--------------------|-----|--------------------|
| A | 2.8 | 3.3 |
| В | 1.0 | 1.2 |
| С | .7 | .8 |
| D | .7 | .7 |



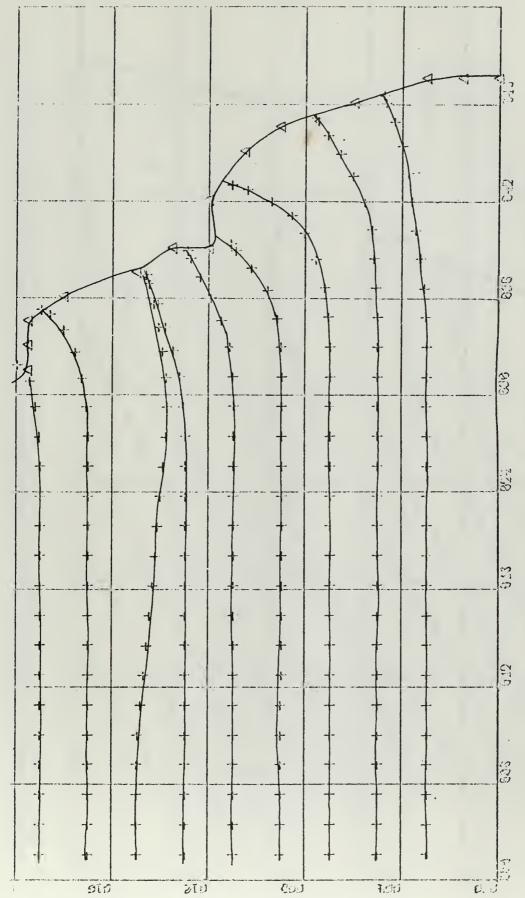


Figure 4. High tide refraction diagram for Big Sur, California. T = 8 sec. $A = 270^{\circ}$. TIME = 30 sec. Latitude $36^{\circ}14'30''$ to $36^{\circ}17'42''$. East-West scale 6000 Ft./inch. North-South scale 4000 Ft./inch.

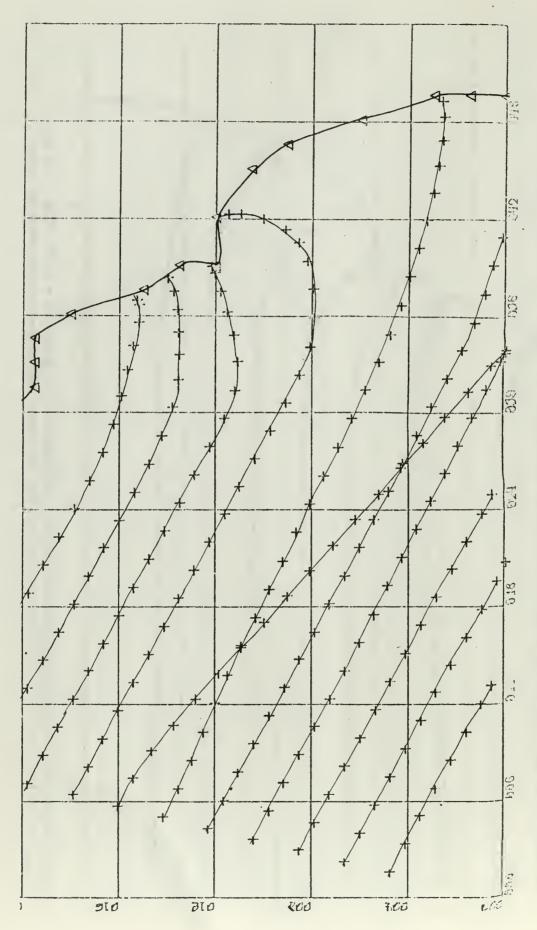


Figure 5. Low tide refraction diagram for Big Sur, California. T = 8 sec. $A = 310^{\circ}$. TIME = 30 sec. Latitude $36^{\circ}14'30''$ to $36^{\circ}17'42''$. East-West scale 6000 Ft./inch. North-South scale 4000 Ft./inch.

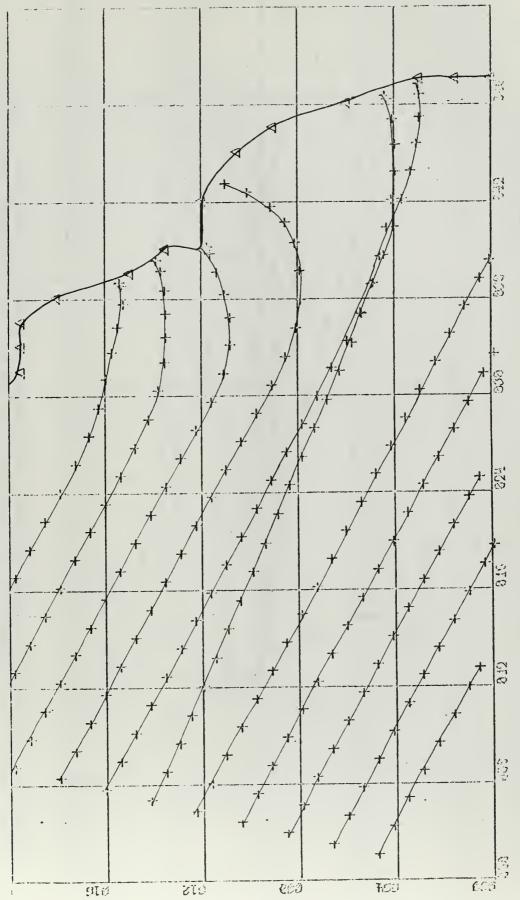


Figure 6. High tide refraction diagram for Big Sur, California. T=8 sec. $A=310^{\circ}$. TIME = 30 sec. Latitude $36^{\circ}14'30''$ to latitude $36^{\circ}17'42''$. East-West scale 6000 Ft./inch. North-South scale 4000 Ft./inch.

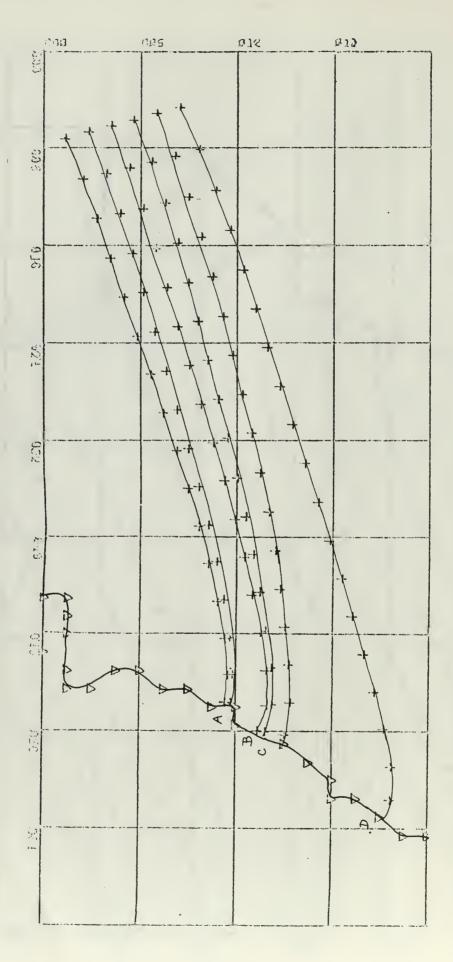


Figure 7. Low tide refraction diagram for Eastern Head, Maine. T = 16 sec. A = 070° . TIME = 30 sec. Latitude $44^{\circ}39'35"$ to latitude $44^{\circ}44'45"$. East-West scale 8000 Ft./inch. North-South scale 6000 Ft./inch.

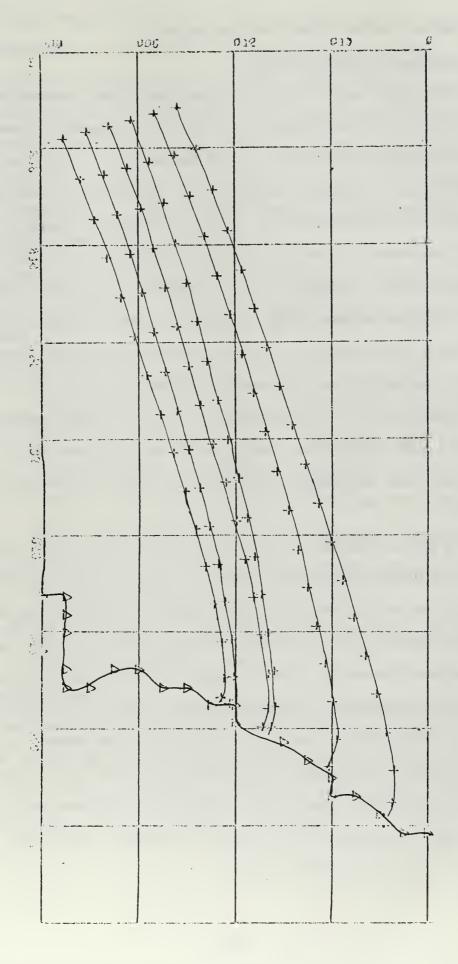


Figure 8. High tide refraction diagram for Eastern Head, Maine. $T=16~sec.~A=070^{\circ}$. TIME = 30 sec. Latitude $44^{\circ}39'35"$ to latitude $44^{\circ}44'45"$. East-West scale 8000 Ft./inch. North-South scale 6000 Ft./inch.

4. Bottom Friction Effects on Wave Height.

Introduction to bottom friction.

The present method for calculating wave height assumes that the propagation of energy between two adjacent orthogonals is conserved as the wave proceeds from deep water to shallow water. This implies that the addition of energy by surface winds, dissipation of energy by bottom friction or internal friction, and the reflection of energy by a steep slope are absent. In nature this is not the case - the wave energy between adjacent orthogonals may be changed by all of these means.

According to Lamb (1932), in fluids of small viscosity (such as sea water), the velocity of the wave and the relationship between depth, length, and period are unaltered by internal friction. This is based on the assumption of a sinusoidal wave form but the result appears to indicate a very slow rate of damping of all waves in deep water, which is in accord with the limited observations in nature (Putnam and Johnson, 1949).

At the sea bottom, however, friction can cause a loss of wave energy. It can be demonstrated both theoretically and experimentally that the particle velocities of shallow water waves are finite at the bottom and actually approach the theoretical orbital values at a short distance from the bottom. Therefore, a boundary layer in which mechanical energy is dissipated through the distortion of fluid elements must exist at the bottom. The effective roughness of a sandy bottom probably is determined by the size of the ripples in the sand rather than by the size of the sand grains. The ripple size, however, appears to be determined by the size and sorting coefficients of the sand in addition to the height and period of the wave and the depth of water.

Putnam and Johnson established the general magnitude for the dimensionless friction coefficient for the sea bottom to be 0.01. (This value was computed assuming initial conditions of 12 second waves, 4 feet high, traveling over a sandy beach with a ripple spacing of 5 inches.) Putnam and Johnson point out that since the friction factor only enters the equations for wave height to the first power, the friction factor could change by \pm 50 percent without appreciably changing the order of magnitude of wave-height reduction.

Based on the following assumptions Putnam and Johnson were able to develop a procedure to calculate wave height when bottom friction is included.

- (1) The oscillatory motion at the bottom is sinusoidal and corresponds to the wave theory based on small amplitudes.
- (2) The friction coefficient does not vary with velocity and is approximately independent of water depth. This assumption amounts to choosing an average friction factor for the entire region over which the sea bottom affects the wave motion.
- (3) Perpendicular flow at the bottom resulting from percolation in the bottom material is negligibly small.
- (4) The sea bottom has a constant upward slope from offshore to the breaker point.

Computing procedures.

Bretschneider and Reid (1954) extended the original work of Putnam and Johnson and developed new equations to solve for wave height which included simplified equations for the special cases of constant water depth and constant bottom slope.

Bottom friction is of importance only on gently sloping beaches and in shallow water; thus it is possible, when using numerical wave refraction procedures, to calculate the reduction to wave height due to bottom friction by making the assumption that the water depth is constant between successive computations along the ray path from deep water to the shoreline.

The wave height at each step is computed by the equation

$$H = H_{1} \left[1 - \left\{ (1 - K_{f_{1}}) + (1 - K_{f_{2}}) + \cdots + (1 - K_{f_{m}}) \right\} \right]$$
 (4)

where

 H_1 = the calculated wave height, neglecting bottom friction, for the point:

 K_{f_n} = the refraction factor between point n-1 and n. The coefficient of friction (K_f) is calculated by the equation (from Bretschneider and Reid)

$$K_{f} = \frac{1}{A' p_{f}} \Delta \chi + 1$$

$$A' = \frac{f H_{1}}{T^{4}};$$

$$p_{f} = \frac{64 \pi^{3}}{3g^{2}} \left(\frac{K_{S}}{\sinh \frac{2 \pi \alpha}{L}}\right)^{3};$$

$$K_{S} = \text{direct shoaling factor;}$$

$$\Delta \chi = \text{incremented distance.}$$
(5)

 $K_{\mathbf{f}}$ is computed at each ray point with Δx equal to the distance along the wave ray between point n and n + 1. It is assumed in the computations that the water depth between point n and n + 1 is constant and equal to the depth at point n.

Results of bottom friction computations.

The accuracy of the approximation that water depth is constant can be seen by a comparison of wave height obtained by this procedure to the wave height obtained by the more sophisticated and accurate procedure used by Putnam and Johnson. This comparison is shown in Table 3 (computed for wave period = 12 seconds, bottom slope = 5 feet). The comparison is very favorable along the entire distance of travel from the point that the wave first feels bottom to the breaking point. The difference is certainly within the order of magnitude of the error introduced by other assumptions. The difference in values is caused by the approximation to constant depth steps in the calculating procedure.

Since this program uses constant time steps in moving each ray point toward the shoreline, the horizontal distance over which depth is assumed constant can be reduced by shortening the computing time interval (TIME). Table 4 shows the reduction (in percent) of wave height at the breaking point for different computing time intervals (with wave and bottom conditions the same as shown for Table 3 above). As expected, there is a slight increase in wave height reduction as the computing interval is decreased. It can be concluded that the value of TIME will affect the wave heights calculated by this procedure, but the effect is small.

Realizing the limitations imposed by the initial assumptions of Putnam and Johnson and the calculating procedure employed here, the computer program was used to compute the wave height (with bottom friction included) for varying initial periods and bottom slopes. The results are shown in Table 5. (In arriving at these results it was assumed that the initial wave height was 5 feet and the friction coefficient was 0.01.)

The reduction in wave height (in terms of percent of the wave height

TABLE III

WAVE HEIGHT REDUCTION DUE TO BOTTOM FRICTION AS DETERMINED BY PUTNAM
AND JOHNS'ON AND BY THE NUMERICAL WAVE REFRACTION PROGRAM.

T = 12 sec. $H_0 = 5 \text{ FT.}$ f = .01 TIME = 30 sec. Bottom slope = 1:300 Breaking depth = 11.5 Ft.

| DISTANCE FROM WHERE h = Lo/2 TO BREAKING POINT | PERCENT REDUCTION FARRELL CONSTANT DEPTH STEPS | WAVE HEIGHT PUTNAM & JOHNSON CONSTANT SLOPE |
|------------------------------------------------------|------------------------------------------------------|---------------------------------------------|
| PCT | PCT | PCT |
| 0.0 | 0.0 | 0.0 |
| 50.0 | 0.0 | 0.0 |
| 62.0 | 0.2 | 0.2 |
| 70.3 | 0.4 | 0.4 |
| 80.5 | 1.0 | 1.1 |
| 86.8 | 1.9 | 2.1 |
| 90.8 | 3.1 | 3.2 |
| 93.0 | 4.2 | 4.5 |
| 95.0 | 6.0 | 6.2 |
| 97.1 | 8.7 | 9.3 |
| 100.0 | 18.5 | 20.8 |

| TIME (SEC) | WAVE HT. REDUCTION (%) |
|------------|------------------------|
| 20 | 19.0 |
| 30 | 18.5 |
| 40 | 18.0 |
| 50 | 17.0 |

TABLE V

WAVE HEIGHT REDUCTION FOR BOTTOM SLOPES OF 1:100 to 1:400 AND WAVE

PERIODS OF 8 to 16 SECONDS (PERCENT)

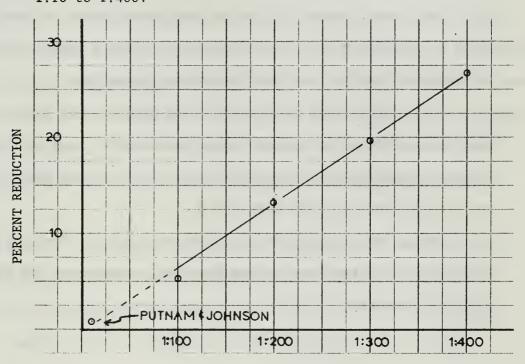
| WAVE | | BOTTOM | SLOPE | |
|---------|-------|--------|-------|-------|
| PERIOD | 1:100 | 1:200 | 1:300 | 1:400 |
| 8 | 5.4 | 14.0 | 21.5 | 28.6 |
| 10 | 5.2 | 13.5 | 20.2 | 27.0 |
| 12 | 5.2 | 13.0 | 18.5 | 24.8 |
| 16 | 5.0 | 12.2 | 18.2 | 24.2 |
| AVERAGE | 5.2 | 13.2 | 19.6 | 26.9 |

without friction) is shown for the theoretical breaking depth (from H.O. 234). To obtain the percent reduction at the specific breaking depth, it was necessary to fit a smooth curve to the plotted points of wave height reduction at the three closest water depths and interpolate the wave height reduction at the breaking point.

Figure 9 shows the average reduction (for all periods from Table 5) of wave height versus bottom slope. The curve is extended by a dashed line to the reduction amount found by Putnam and Johnson for a 1:10 beach slope.

The quantitative value of Figure 9 is controlled by the input accuracy of the coefficient of friction, the initial assumptions and the computation procedure. However, Figure 9 does show, qualitatively, the effect of bottom slope on wave height reduction due to bottom friction. Figure 9 is considered sufficiently accurate to be used in surf forecasting on sandy beaches to provide a first approximation of wave height reduction due to bottom friction.

Figure 9. EXPECTED WAVE HEIGHT REDUCTION AT BREAKING POINT DUE TO BOTTOM FRICTION FOR BOTTOM SLOPES OF 1:10 to 1:400.



BOTTOM SLOPE

5. Conclusions.

The numerical approach to wave refraction is a powerful tool in that it eliminates human subjectivity and permits the refraction computations to account for more factors than is possible with hand drawn techniques.

A computer program that uses constant time intervals between computations has more suitability in accounting for the effects of friction, wind, reflection, etc., than a computer program that employs constant distance steps.

Tide level changes can have a significant effect on wave refraction.

This is especially true in areas where there is a large tide range or an offshore bar that can cause selective refraction.

The changes shown by the tide level effects also point to the importance of having an accurate water depth grid. To avoid significant errors in the refraction pattern a great deal of care must be exercised in the preparation of the depth grid.

Bottom friction can and does have an effect on the wave height.

The numerical refraction program provides a convenient way to account for this reduction.

6. Recommendations.

The course of this investigation has permitted a close look at wave refraction from many points of view. The numerical refraction program is capable of yielding wave refraction information with great speed and accuracy. (Computer time is approximately 5 minutes for a 20 ray diagram.) However, before the advantages of the computer product can be fully realized the author believes further investigations and improvements are needed in the areas of input data, computing procedures and field investigations.

The need for accurate depth information is essential to the computation of accurate refraction patterns. It is anticipated that the present efforts of the Naval Oceanographic Office and the Coast and Geodetic Survey in the area of color photography will improve the water depth information especially in areas where the bottom contours change considerably and in the shallow areas where it is difficult for hydrographic vessels to proceed.

The computer program can be expanded to account for more variables and to provide additional output information; in particular:

- (1) The program should account for the fact that in nature there usually exists a spectrum of wave periods and directions, rather than the single period and direction that is considered here.
- (2) For amphibious operations it would be advantageous if the program determined the breaking depth of the waves, printed out the height of the waves at the breaking point, and then contoured the breaking point on the output refraction diagram.

- (3) Another desirable output for amphibious operations is a computer plot on the output diagram of the area where the wave height is a minimum, as this would be an ideal area to anchor large ships.
 - (4) The various subroutines should be checked to ensure that each section is providing comparable accuracy.

Field work is necessary to determine the value of the bottom friction factor for various type bottoms. This is essential before accurate wave height information can be obtained. Field investigations are also needed to determine the accuracy of the computer program; until now the numerical wave refraction procedures have been checked only by comparison with hand drawn results.

- Y + 1,3

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APPENDIX 1

Computer Program for Wave Refraction using Fortran 63 on the C.D.C. 1604 Computer

Subroutine Title: FRICT1

Variables of Subroutine:

RLO.....Deep water wave length.

RLQ......Wave length at point X, Y.

CFF......Friction factor between point n and n+1.

RCFF.....(1 - K_f)

TRCFF......Total of (1-K_f) terms to point n.

TCFF..........Total friction factor to point n.

Summary of Subroutine:

The subroutine calculates the value of K_f and wave height reduced by bottom friction (HHFR) for each point along wave ray.

See Stouppe (1966) for discussion of remainder of program.

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| HOALING WATER BY LT. D. STOUPPE,OCTOBER 1966 PROGRAM FOR THE COMPUTATION OF CREST AND RAY MODIFIED BY LT. C. FARRELL, MAY 1967, TO ECT OF FRICTION AND TIDE LEVEL. THE PROGRAM IS RAN 63 FOR THE CDC 1604 COMPUTER SYSTEM. THE DESIRED NAMES FOR THE GRAPH TITLES IN THE TS AS DESCRIBED IN THE WRITEUP ON THE DRAW E(12),XXX(100),YYY(100),FX(100),FY(100) AA(100) AA(100) *CXY,PAR,BAR,TIME,GRID,FK,MAX,NOR,MM,NN,DEPTH,HA. | OO), PDPX, PDPY, PCPX, PCPY, PDDPXX, PDDPYY DLO,C(6), MIT, DEL X, DEL Y, FCF, TIDE, HINT, DEPTH1 RIGHT ADJUSTED NN), I=1, MM) HINT, T, A1, NOR, TIME, DIST, GRID | 2HXV,5X,2HYV,4X,3HFCF,1X,4HTIDE,1X,4HHINT,2X,3HNOR,2X,4HGRID) CF,TIDE,HINT,T,A1,NOR,TIME,DIST,GRID \$ DO 516 J=1,NN \$516,502 J)*6.0-TIDE |

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| \$ BAR=6.283185/T ITLE(2)=8H WAVE RE \$ A=A1 | V=XV-DIST*SINF(A) \$ YV= YV+DIST*COSF(A) \$XX=XV \$ YY=YV \$ GO TO 3 ORA=1 X=X(NORA) \$ YY=Y(NORA) \$ A=AAA(NORA) ALL RAYN(XX,YY,NOR) ONTINUE \$ NORA=NORA+1 \$ IF(NORA-NOR)10,10,4 = (MAX-1)5,5,24 = (MAX-1)20,20,25 = (NIP-4)21,26,26 = (NIP-4)21,26,26 ALL DRAW(NOR,X,Y,1,2,LABEL,ITITLE,100000,100000,0,0,0,2,2,9,9,1) ALL DRAW(NOR,X,Y,2,2,LABEL,ITITLE,100000,100000,0,0,0,2,2,9,9,1) ALL DRAW(NOR,X,Y,2,0,LABEL,ITITLE,100000,10000,0,0,0,2,2,9,9,1) ALL DRAW(NOR,X,Y,2,0,LABEL,ITITLE,100000,10000,0,0,0,2,2,9,9,1) |

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| • AAA(100) • A•CXY•PAR•BAR•TIME•GRID•FK•MAX•NOR•MM•NN•DEPTH•HHA• 0•DIST | DEP(100,100),PDPX,PDPY,PCPX,PCPY,PDDPXX,PDDPYY CC,PDPC | FFKK(100), DLO, C(6), MIT, DEL X, DEL Y, FCF, TIDE, HINT, | K=XR+1.5 \$ YR=YY/GRID \$ L=YR+1.5 | IF((XX-1.0)*((MM-1)-K))62,7,7 IF((YY-1.0)*((NN-1)-L))62,6,6 | 091=1,6 | E(I)=0.0 | +1 \$ N=K-1 \$ NA=K+1 | \$ DOIO I=N*NA \$ O(I)=(I-1)*GRID \$ P(J)=(J-1)*GRID |)*P(J)\$D(5,1)=D(5,1)+O(1)**25D(6,1)=D(6,1)+P(J)**25E(1)=E(1)+DEP | (I,J)\$D(1,2)=D(2,1)\$D(2,2)=D(5,1)\$D(3,2)=D(4,1)\$D(4,2)=D(4,2)+O(I | 1**2*P(J)\$D(5,2)=D(5,2)+O(I)**3\$D(6,2)=D(6,2)+O(I)*P(J)**2\$E(2)= | C.Z.+O(1)*DEP(1•3)*D(1•3)*D(3•1)*D(3•1)*D(4•1)*D(4•1)*D(3•3)*D(6•1)*D(4• | 1) | 4)=D(4,4)+O(I)**2*P(J)**25D(5,4)=D(5,4)+O(I)**3*P(J)5D(6,4)=D(6,4) | 1+O(I)*P(J)**35E(4)=E(4)+O(I)*P(J)*DEP(I•J)\$D(1•5)=D(5•I)\$D(2•5)= D(6.2)*P(3.6.1-D(4.2)*P(4.6.2-D(4.6.4))*D(1.6.1-D(4.6.4))*D(1.6.1-D(4.6.4))*D(2.6.1) | D(4*4)8E(5)=E(5)+O(1)**0*DEP(1,1)*D(5*4)*D(5*5)=D(5*5)=D(6*5)=D(4*4)8E(5)=E(5)+O(1)**0*D(4*5)=D(4*4)8E(5)=E(5)+O(1)**0*DEP(1,1)*D(1,4)=D(4*4)8E(5)=E(5)=E(5)+O(1)**0*DEP(1,1)*D(1,4)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4*5)=D(4 | \$6)=D(6*3)\$D(4*6)=D(6*4)\$D(5*6)=D(4*4)\$D(6*6)=D(6*6)+P(J)**4\$E(6) | 3=E(6)+P(J)**2*DEP(I•J) | 34M=1*6 \$ KD=0 \$ 7-0 \$ 0.000 \$ 7-17. | 1) 11,12,12 | 1 | \$Z=D(M,J) \$ D(M,J)=D(KP,J) | 0 IF(ABSF(D(M,M))00001)50,50,30 | |

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              LPI=M+1 $ D034N=LP1,6 $ IF(D(N,M))32,34,32
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                                                                                                                                                                                                                                                                                                                       DO6 M=1,50 $ CXX=PAR*IANHF(BAR*DEPTH/CXY)
                                                                                                                                    C(II) =(D(II, JPN)-S)/D(II, II) $ KER=1
                                                                             JPN=7
                                                                                                                                                                                                                                                                                          ALO=PAR*T $ DLO=DEPTH/ALO $ IF(DLO
                             RATIO=D(N,M)/D(M,M)$ D033J=LP1,NPM
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       COMMON/BLK1/X(100),Y(100),COREFR(1
                                                                                                                                                                                                                                                                                                                                      IF (ABSF (CXX-CXY)-0.01)61,61,60
                                                                                                                                                                                                                                                                                                                                                                                                                PDDPXY=C(4) $ PDDPXX=2*C(5)
                                                                                                                                                                                                                                                                                                                                                                                   PDPX=C(2)+C(4)*YY+2*C(5)*XX
                                                                                                                                                                                                                                                                                                                                                                                                 PDPY=C(3)+C(4)*XX+2*C(6)*YY
                                             D(N,J)=D(N,J)-RATIO*D(M,J)
                                                                                                        IIP1=II+1 $ DO 42N=IIP1,6
                                                                                                                                                                                                                                                                                                                                                                    $ NGO=2 $ LO=2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        SUBROUTINE KFUNCT (A,FK)
                                                                                                                                                                                                 117H MATRIX SINGULAR/)
                                                                           D043I=1,6 $ II=7-I $
                                                                                                                                                                   PRINT 54,XX,YY,KER
                                                                                                                                                                                                                                                                           IF (DEPTH) 62,62,65
                                                                                                                                                                                                                                                                                                                                                      CXY = (CXX + CXY) * .5
                                                                                                                                                                                                                GO TO (52,53)KER
                                                                                                                      S=S D(II.N)*(N)
IF (M-6)31,40,40
                                                                                                                                                                                                                                                                                                                                                                    CONTINUE
                                                            CONTINUE
                                                                                                                                                                                                                                             CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                GO TO 64
                                                                                                                                                                                                                                                                                                                                                                                                                                                              GO TO 81
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             CONTINUE
                                                                                                                                                                                                                              RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                              NG0=1
                                                                                                                                                    KER=2
                                                                                          143
                                                            34
                                                                          0 4
                                                                                                                      42
                                                                                                                                     43
                                                                                                                                                   20
                                                                                                                                                                                  54
                                                                                                                                                                                                                                                                                                       99
                                                                                                                                                                                                                                                                                          65
                                                                                                                                                                                                                                                                                                                                                      09
                                                                                                                                                                                                                                                                                                                                                                                                                                                                              49
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           81
                                                                                                                                                                                                                                                                                                                                                                    61
                                                                                                                                                                                                                                                                                                                                                                                   29
```

| | 0000234 0000234 0000235 0000238 0000238 0000240 0000241 0000244 0000244 0000244 0000244 0000244 0000247 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1NOGO(100),NGO,AAA(100) COMMON/BLK2/T;UU,CXY;PAR,BAR,TIME,GRID,VV;MAX,NOR,MM,NN,DEPTH,HHA 1,COREFA,NORA,LO,DIST COMMON/BLK3/DEP(100,100),PDPX,PDPY,PCPX,PCPY,PDDPXX,PDDPYY 1,PDDPXY,PDDPCC,PDPC 3 GO TO (5,6)LO 6 R1=CXY/32.2 \$ R2=R1**3*BAR**2 \$ R3=R1**5*BAR**4 \$ R4=R1**7*BAR**6 PDPC=2.*R1+4.*R2/3.+6.*R3/5.+8.*R4/7. PDDPCC=(2.*R1+4.*R2/3.+6.*R3/5.+8.*R4)/CXY PCPX=PDPX/PDPC \$ PCPY=PDPY/PDPC FK=(PCPX*SINF(A)-PCPY*COSF(A))/CXY \$ GO TO 4 5 FK = . 4 RETURN END SUBROUTINE MOVE(X,Y) COMMON/BLK1/U(100),V(100),COREFR(1),HHO(100),B(3,100),FFK(100),COMMON/BLK1/U(100),NGO,AAA(100) COMMON/BLK2/T;A*CXY;PAR,BAR,TIME,GRID,FK,MAX,NOR,MM,NN,DEPTH,HHA, | 1COREFA,NORA,LO,DIST GOMMON/BLK3/DEP(100,100),PDPX,PDPY,PCPX,PCPY,PDDPXX,PDDPYY 1,PDDPXY,PDDPCC,PDPC GOMMON/BLK4/FFKK(100),DLO,C(6),MIT,DEL X,DEL Y,FCF,TIDE,HINT, 1HHF,HHFR(100),LQQ,HHG,DEPTH1 FKBAR=FFKK(NORA) IF(MAX-1)1,1,4 IFKBAR=FK 4 MIT=1 DEL D=TIME*CXY GO TO (22,21)LO 22 XX=X+DELD*COSF(A) \$ YY=Y+DELD*SINF(A) \$ AA=A \$ FKK=0, \$ FKBAR=0, GO TO (22,21)LO 22 XX=X+DELD*COSF(A) \$ YY=Y+DELD*SINF(A) \$ AA=A \$ FKK=0, \$ FKBAR=0, GO TO 6 21 BO 20IT=1,30 19 DEL A=FKBAR*DEL D \$ AA=A+DEL A \$ ABAR=A+,5*DEL A \$ DEL X=DEL D* 1 COSF(ABAR) \$ DEL Y=DEL D*SINF(ABAR) \$ XX=X+DEL X \$ YY=Y+DEL Y GO TO (101,6)MIT 101 CALL DEPTFUN(XX,YY) \$ GO TO(38,10)NGO |

| 00002520 | 0000255 | 0000257 | 00002590 | 0000260 | 00002620 | 0000264 | 0000266 | 0000267 | 0000269 | | | 00002740 | 0000275 | 00002760 | 0000278 | 00002790 | 0000281 | 0000282 | 0000284 | 00002850 |
|-----------------------------------------|------------------------------|--------------------------------|-------------|-----------------------------|--------------------|----------|----------------------------|---------|---------|---------------------------------------------------------------------------------------------|--------------|--------------|---------|----------|--------------|----------|---------|---------|---------|-----------------------------------------------------------|
| CALL KFUNC IF (IT-13)5 FKKPP=FKBA | 5 IF (MAX-1) 7 IF (IT-1)2 | 9 IF (ABSF (FKKP) O FKKP=FKBAR | IF (ABSF (F | 60 TO 38 Frade : 5 + (Fr | 6 NGO=2 \$ GO TO 8 | GONTINUE | FFKK(NORA)=FKBAR BETIEM | | I NE | COMMON/BLK1/X(100),Y(100),COREFR(1),HHO(100),B(3,100),FFK(100), 1NOGO(100),NGO,AAA(100) | COMMON/BLK2/ | COMMON/BLK3/ | PDDF | 3LK4/ | GO TO (5,6)L | • | 60 TO 7 | PDDPY | DDC = | *COSF(A)*PCCPXY+COSF(A)**2*PC *((DEL X)**2+(DEL Y)**2) |

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00002880
                                                          00002910
                                                                           00002920
                                                                                             00002930
                                                                                                                                                                  00002970
                                                                                                                                                                                                                                                                                                                                                                                    00003090
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               00003160
                                                                                                                                                  296
                     0000289
                                        0000290
                                                                                                             0000294
                                                                                                                               0000295
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         0000321
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0000322
                                                    CC0=CXY/PAR $ HSHOL=3.2519-12.8150*CC0+28.8112*CC0**2-29.9257*CC0
                                                                                                                                                             FRICT (HHA, CXY, TIME, DEPTH, LQQ, PAR, T, HHF, NORA, NOR,
B(3,NORA)=(B(1,NORA)*(P*DD-2,)+B(2,NORA)*(4,-2,*DD**2*Q))
                                                                                     CALL FRICT (HHA,CXY, TIME, DEPTH, LOQ, PAR, T, HHF, NORA, NOR,
                                                                                                                                                                                                                                                                                                                                                                                [HET1=64.0*(3.141592**3)/(3.0*(32.2**2))
                                                                     1**3 11.6815*CCO**4 $ HHA=COREFA * HSHOL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          CFF=1.0/((APR*THETA*DEL D/HSHOL)+1.)
                                 COREFA=1./SQRTF(ABSF(B(2,NORA))
                                                                                                                                                                                                 DIMENSION TOFF (100), TROFF (100)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       [HET2=SINHF (6.28315*DEPTH/RLQ)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        [HETA=THET1*((HSHOL/THET2)**3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             RCFF (NORA) = TRCFF (NORA) + RCFF
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               TCFF (NORA) = 1 . - TRCFF (NORA)
                                                                                                                                                                                                                                                                                                                                                                                                 HHQ=HHA*HINT*TCFF (NORA)
                                                                                                                                                                                                                                                     IF(LQQ-1)715,701,715
                                                                                                                                                                                                                                                                                                                                                                                                                                                    APR=FCF*HHQ/TFP
                                                                                                                                                                               FCF, HINT, HSHOL)
                                                                                                        1FCF, HINT, HSHOL)
                                                                                                                                                                                                                                     DEL D=IIME*CXY
                                                                                                                                                                                                                                                                        DO 8 I=1, NOR
                                                                                                                                                                                                                                                                                                                           RCFF(I)=0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  HHF = HHQ / HINT
                                                                                                                                                                                                                   CC0=CXY/PAR
                                                                                                                                                             SUBROUT INE
                                                                                                                                                                                                                                                                                       CFF(I)=1.0
                                                                                                                                                                                                                                                                                                          ICFF(I)=1.0
                                                                                                                                                                                                                                                                                                                                                                                                                                     RLQ=RLO*CCO
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            RCFF=1.-CFF
                  1/(2.+P*DD)
                                                                                                                                                                                                                                                                                                                                                                                                                    RLO=PAR*I
                                                                                                                                                                                                                                                                                                                                            CONTINUE
                                                                                                                                                                                                                                                                                                                                                               PP=1**4
                                                                                                                          RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   LQQ=5
                                                                                                                                             END
                                                                                                                                                                                                                                                                                                                                                                                                  715
                                                                                                                                                                                                                                                                     701
                                                                                                                                                                                                                                                                                                                                              00
```

| | ZI | | SINHF | | |
|----------|-----------------|----------|----------------|----------------------|-----------|
| | DYS-EM FNTDV | | <u> </u> | | |
| ЯХР | _ | | 000 | | SINHF 003 |
| ے م | ה ל ה | | 700.7 | | 2 9 |
| _ م | 000 | | 00054271027757 | | |
| EXP3 | 100 | | 0000000 | | SINHEOON |
| ۵ | 00.1 | | 00256125073122 | | |
| ۵ | 001 | | 77343354634402 | | |
| ۵ | OCT | | 00370456067701 | | |
| ۵ | OCT | | 00753562056056 | | |
| ۵ | 100 | | 0- | | 7 |
| ш. | DEC | | 1.0 | | 7 |
| <u>ب</u> | DEC | | 0 • 0 | | 1 |
| SINHF | SLJ | | * | | |
| | SIU | _ | SINH3 | | |
| | LIU | ~ | SINHF | | 1 |
| | LDA | _ | 0 | | |
| | ARS | | 24 | | |
| | SAU | | SINH1 | STORE ARGUMENT | SINHFOZO |
| | INI | ~ | _ | | 2 |
| | SIL | _ | SINH3 | STORE RETURN ADDRESS | 70 |
| SINHI | LDA | | ** | | 7 0 |
| | AJP | | SINH3 | EXIT IF ZERO | 7 |
| | THS | | EXP | CK MAXI | 2 |
| | SLJ | | ERROR1 | | 2 |
| | THS | | EXP1 | CHECK MINIMUM X | 2 |
| | SLJ | | *+2 | |)2 |
| | SLJ | | ERROR1 | |)2 |
| + | FDV | | EXP2 | | SINHF03 |
| | STA | | =STEMP | | 3 |
| | FSB | | XP3 | | 3 |
| | SAL | | SINHS | | |
| | FAD | | EXP3 | | 3 |
| | 7 V C | | ₩ | | 33 |
| | 4 - N | | I EMP | | 33 |

| TEMP =STEMP1 =STEMP2 =STEMP2 EXP6 TEMP2 EXP5 | TEMP2 TEMP1 TEMP1 TEMP1 | 36 36 ** *+1 -1 | 36 ERRORI TEMP ONE TEMP EXP8 | SINH3 ** ERROR2 QBQIBJOB ISINH OUT QBQEXITS |
|----------------------------------------------------------------|----------------------------------|-------------------------------------------------------|-------------------------------------------------------------------|---------------------------------------------|
| | | 7 | m | 9 |
| S T A D A D A D A D A D A D A D A D A D A | STAD STAD FSBAD FOV | I A I I R S I R P A P A P A P A P A P A P A P A P A P | S L L S A L L S A C L S A C L S C C C C C C C C C C C C C C C C C | SLU SLU CALL CALL 51 |
| | | SINH2 | | SINH3 ERROR1 ERROR2 |

SINHF054 SINHF055 SINHF056 SINHF057 SINHF058 SINHF059 SINHF06

SINHF062 SINHF063 SINHF064 SINHF066 SINHF067

SINHF065

SINHF069 SINHF07

SINHF068

SINHF071 SINHF072

SINHF051 SINHF052 SINHF053

SINHF041 SINHF042 SINHF044 SINHF045 SINHF046 SINHF047 SINHF047 SINHF047

SINHF037 SINHF038 SINHF039

SINHF04

| 2 IF (JT.EQ.1) 51,53 3 IER=2 \$ IBCD=202020202020208 \$ GO TO 60 4ECK IF INPUT IS MORE THAN EIGHT PLACES 1 IF (NX) 5,6,7 7 NEG=1 8 IF (NX.GT.9999999) 10,17 9 NT=99999999999999999999999999999999999 | 000002 000003 000004 000005 000000 000010 00011 000112 000114 000115 000116 000116 |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| =9 I N(J)=MODF(FTEN,10.) TEN=INTF(FTEN/10.) D TO 25 NPUT IS ZERO SET ALL BLANK EXCEPT NN(8)=0 D 40 I=1.7 N(I)=BLANK N(B)=12B BL=7 D TO 21 D 3 I=1.8 F(NN(I).EQ.0) 4.8 N(I)=BLANK D 3 I=1.8 F(NN(I).EQ.0) 4.8 | |

```
00034
                  00036
                                            000039
                                                                                                                                           00050
00051
00052
53
                                   00038
                                                                       42
                                                                                                00045
                                                                                                                 00047
                                                                                                                          00048
                                                                                                                                  6400FO
                                                                               00043
                                                                                        00044
                                                                                                         00046
                                                              00041
                                                                                                                                                  ENCODE(8,50,1BCD) (NNT(I),1=1,8)
                                         IF (NEG.EQ.0) 20,21
NN(JBL)=408 $ JBL=JBL-1
IF (NP-JBL)26,26,27
       BO 41 KK=I,8
IF(NN(KK),EQ,0) 42,41
                                                                            IF(JT.EQ.0) 30.31
KJ=NP $ GO TO 32
                                                                                                                                         NNT (K+8-KJ) = NN (K)
                                                                                                              DO 33 K=NK.8
NNT(K-KJ)=NN(K)
DO 34 K=1.KJ
                                                                                                                                                          FORMAT(8R1)
                         NN(KK)=12B
                                                                                              KJ=JBL-NP
                                  CONTINUE
                                                                                                       NK=KJ+1
JBL=I-1
                                                                   = dN
                         42 7
                                                   20
21
27
30
                                                                                                                         33
```

APPENDIX 2

Sample Computer Input and Output

Sample Input

52 56

```
440 • 460 • 515 • 505 • 510 • 470 • 425 • 355 • 300 • 330 • 370 • 360 •
340 · 345 · 350 · 365 · 375 · 390 · 455 · 560 · 700 · 850 · 950 · 865 ·
550 • 490 • 470 • 460 • 415 • 380 • 335 • 400 • 460 • 520 •
                                                           560.
                                                            53.
510 • 460 • 400 • 315 • 275 • 300 • 220 • 140 •
                                                80.
                                                      55.
                                                                  51.
360 • 405 • 450 • 435 • 445 • 380 • 330 • 270 • 235 • 250 •
                                                           280.
290 • 297 • 300 • 305 • 315 • 360 • 420 • 490 • 655 • 810 •
                                                           930.
540 • 500 • 515 • 515 • 480 • 445 • 440 • 500 • 540 • 593 • 630 • 630 •
 610. 560. 515. 400. 315. 275. 234. 190. 110.
                                                            53.
                                                      63.
 295 • 370 • 370 • 325 • 325 • 307 • 260 • 190 • 170 • 175 •
                                                          190. 190.
 240 • 250 • 250 • 265 • 275 • 305 • 405 • 470 • 570 • 750 •
                                                          900. 885.
 560 • 540 • 550 • 560 • 550 • 540 • 550 • 605 • 620 • 640 • 610 • 570 •
 540 · 590 · 570 · 430 · 300 · 310 · 280 · 240 · 170 ·
                                                      70.
                                                            55. 50.
240 • 250 • 235 • 195 • 225 • 200 • 190 • 160 • 130 • 140 • 160 • 170 •
190 • 195 • 210 • 230 • 250 • 280 • 350 • 427 • 490 • 670 • 820 • 860 •
610 • 590 • 590 • 615 • 600 • 610 • 620 • 660 • 650 • 620 • 560 •
480 · 520 · 560 · 515 · 405 · 370 · 320 · 260 · 190 ·
                                                     80.
                                                           60.
                                                                 49.
150. 195. 200. 110. 125. 120. 115.
                                         95.
                                               80.
                                                     98. 105.
                                                                110.
           180 • 205 • 210 • 210 • 290 • 380 • 455 •
                                                          740.
162.
     170.
                                                    580.
660. 650. 640. 660. 660. 670. 690. 685. 650.
                                                    585.
                                                          510.
                                                                480.
450. 490.
           550. 525.
                       450. 380. 325. 270. 215.
                                                    120.
                                                           65.
                                                                 50.
                 75.
                       73.
                                        70.
80.
     80.
           90.
                             70.
                                   72.
                                              70.
                                                    70.
                                                          72.
                                                                80.
110. 120.
           140 • 160 • 175 • 195 • 240 • 290 • 390 • 475 • 610 •
           715. 725.
                       730.
                            710. 680. 655. 620. 540. 480.
                                                                445.
760.
     720.
440.
    410.
           550 • 490 • 415 • 370 • 325 • 285 • 235 •
                                                   170. 85.
                                                                65.
           60.
                 58.
                       60.
                             60.
                                   61.
                                        63.
                                              60.
                                                    60.
                                                          65.
                                                                65.
61.
     60.
            100 . 120 . 130 . 120 . 160 . 230 . 280 .
                                                    380. 460.
                                                                540.
80.
      85.
                       665. 635. 620. 610. 570. 485. 440.
           735. 715.
                       445. 375. 315. 295. 245. 190. 110.
425. 490.
           550. 500.
                             54.
                                   53.
                                         54.
                                              56.
47.
     47.
           48.
                 49.
                       52.
                                                    59.
                                                          60.
                                                                60.
                       82.
                             85.
                                   110. 150. 185. 250.
                                                                395.
63.
            70.
                 77.
                                                          340.
560. 610. 590. 600. 570. 540. 525. 520. 500. 460. 410.
                                                               380.
420. 510. 515. 500. 465. 410. 283. 265. 235. 210. 145. 105.
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^{4500 · 2 · .01 0 · 5 · 0 18 · 0 10 · 0 10 30 · 0 1500 · 0 1500 · 0 4500 · 2 · .01 6 · 5 · 0 18 · 0 10 · 0 10 30 · 0 1500 · 0 1500 · 0}

.9316 HHFR 0 7319.3 1. COREFA HH HHFR C 1.300 1.3000 1.0000 1. .000 .4214 1.0000 1.0000 1.1300 1.7000 1. 300 1.000 1.,000 1.000 1.0000 9336 1.0000 1.0000 . A HH. FA HH .9316 1.0000 1.0000 1.000 1.000 1.2019 1.0885 1.3000 1.3003 1.3050 1.0000 1.3030 1.0000 1.0.00 1.000 .9214 .9396 .9093 1.3038 1.1000 COREFA 1. 838 1.0.00 1.300 1.0000 1.1000 1.0000 1.0000 1.000 1.0000 1.1.38 1.0000 1.0003 1.500 1.1000 76 4.8 7678.5 83.4.1 7478.7 11418.7 8622.6 11253.5 10594.0 13273.7 11739.0 10754.2 9288.9 8464.2 9517.9 11578.9 8762.2 964).9 9449.1 10433.9 4352.4 2321.6 4387.5 4924.0 3225.7 2148.0 2535.5 3617.6 3443.9 1800.7 1627.3 4526.9 1974.3 3056.4 2882.8 2739.1 3954.7 3792.6 4837.7 4669.1 XX YV FCF TISE HINT T A1 NOR TIME DIST 4500 20000 . 1 (5.0 18.0 10.3 19 33.0 1500 DEPTH NG0 350.2 2 1112.9 2 359.4 2 2 2 2 ~ 2 DEPTH 405.2 943.6 363.2 365.6 1288.8 1181.9 1537.5 1984.2 3351.2 1637.4 383.7 435.9 1117.9 397.4 1124.6 2594.4 20503 1.3020 1.0000 1.3630 .9195 .9179 1.7603 0000.1 1.0000 .9293 .9170 1.0000 1.0000 1.0000 . 3C61 .9129 1.0000 1.3003 1.0000 1.000°. .9195 . 9203 .9413 1.0000 1.3000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 .9170 1.0000 1.0000 .9129 .9293 .9273 .9012 COREFA 1.0000 COREFA 1.0000 CUREFA 1.0156 CUREFA 1.0415 1.000) 1.0000 1.0000 1.0000 1.0003 1.0000 1.0000 1.000. 1.000. 1.0000 00001 0000.1 1.3222 .9928 1.9000 1.000 Y 6826.7 7180.2 7085.7 8196.5 7811.7 6972.2 10242.5 11246.6 9781.3 7971.8 10926.3 9115.9 8237.5 10766.1 9.9568 9941.5 8117.6 10101.6 11086.4 9258.8 NUMBER OF CREST = NUMBER OF CREST = NUMBER OF CREST = NUMBER OF CREST 4988.6 2408.4 3309.2 X 4158.6 3704.4 4439.2 2234.8 2061.2 1887.5 1713.9 3143.2 2969.6 2796.0 2622.3 4027.3 3872.6 4866.2 4729.3 4596.8

357.4 Z

371.4

370.

1685.6

368.1 2

468.5

1578.3

2752.

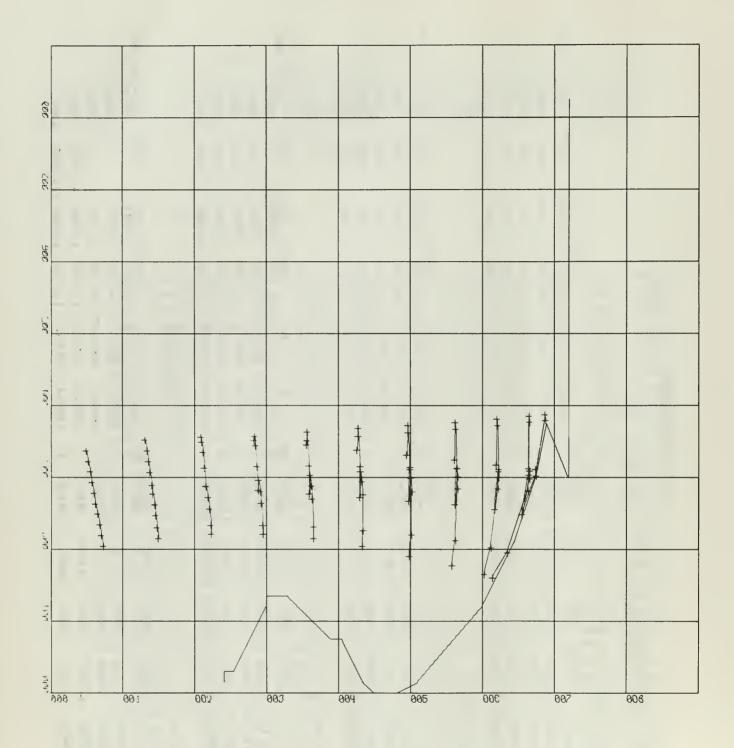
411.3

DEPTH NGO

1396.7 1258.6 2165.4 3411.9

1229.1 2

1413.1 1659.9 2514.5 3891.5



K-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 1.00E+04 UNITS/INCH.

FÄRRELL WAVE REFRACTION PROGRAM MONTER EY BAY

TIDE LUL 0 ANGLE = 0 PERIOD = 18

APPENDIX 3

SUMMARY OF EQUATIONS

Wave Speed (C)

$$C = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{TC}\right)$$
.

Ray Curvature (FK)

Coefficient of Refraction (Kr)

The value of ray separation (eta) is calculated by solving the second order, non-linear differential equation:

$$\frac{D^2 \beta}{Ds^2} + P \frac{D \beta}{Ds} + 9 \beta = 0$$

where

The above equation is solved by the finite difference method. This results in an equation for the Beta value at the n+1 point in terms of the Beta values at the two previous points. The equation to be solved is then:

$$B_3 = \frac{(PD-2)B_1 + (4-2gD^2)B_2}{2+PD}$$

where

D is the incremented distance

p,q are as defined above

 ${\cal B}_{1},\,{\cal B}_{2}$ are the Beta values of the two previous points. The The coefficient of refraction is calculated by the relation:

Direct Shoaling Coefficient (Ks):

$$H_s = 3.2519 - 12.8150(\frac{c}{c_o}) + 28.8112(\frac{c}{c_o})^2 - 29.9257(\frac{c}{c_o})^3 + 11.6815(\frac{c}{c_o})^4$$

Wave Height Reduction Factor Due to Bottom Friction (Kf):

$$K_{\xi} = \frac{1}{\frac{A' \mathscr{O}_{\xi}}{K_{s}}} \Delta H + 1$$

where

$$A' = \frac{f H_1}{T^4}$$

$$\emptyset_f = \frac{64 \pi^3}{35^2} \left(\frac{K_s}{sunh 2 \pi d} \right)^3$$

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| ABSTRACT | | | |

Numerical wave refraction programs permit a detailed study of the transformation of wave energy as waves move from deep water to shallow water. By eliminating the subjectivity that is present with hand drawn diagrams the effect of small variations in the initial assumptions and wave conditions can be investigated. The effects on wave refraction of tide level changes and bottom friction are investigated here. It is demonstrated that a uniform increase in water level as would be caused by tidal fluctuations can cause a significant change in the wave refraction pattern for a given nearshore region. A computing procedure is developed to permit numerical refraction programs to account for bottom friction. The reduction in wave height caused by bottom friction depends primarily on bottom slope; this relation is shown in tabular and graphical form.

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| Numerical Analysis | | | | | | | |
| Bottom Friction Tide level Changes | | | | | | | |
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